A Study of Directional Element Connections for Phase Relays

W. K. SONNEMANN
MEMBER AIEE

THE subject of the performance of single-phase directional relay elements for phase fault protection has never been completely analyzed with consideration given to all the possible variables. The analyses which have been made have been limited in scope when intended to be general in nature, or have been slanted toward specific problems. It is the purpose of this paper to extend the boundaries of the generalities of the analysis in the hope that the results will be beneficial to the industry through making answers available which have heretofore been difficult to obtain.

Scope

The relay directional elements to be studied in this paper are single-phase elements only as normally applied for the purpose of making directional discriminations for interphase faults. Polyphase directional relays are excluded on the basis that the method of approach used here is not adaptable to predicting their performance. Ground relays are excluded on the basis that they may be more conveniently studied as a separate problem, free of many of the complications that enter into a consideration of phase relays. The single-phase directional elements that are to be studied are used widely and thus form a conveniently separable bloc in the over-all problem of directional discrimination. This bloc is characterized by certain conditions that may be grouped into the two classifications of (1) fixed, and (2) variable considerations. These are as follows:

Fixed Considerations

There are three relay directional elements for a 3-phase system, one for each phase. There are four types of faults, which are phase-to-phase, single-phase-to-ground, 2-phase-to-ground, and three phase. There are four types of connections which are commonly used, these being the 90-degree, two forms of 60-degree, and the 30-degree connections. (These will be further described.) There is also the consideration of the location of the relays in the power system relative to delta star (or star delta) connected transformer banks, parallel feeds, and zero sequence sources. The author believes that the variables of location in the system can be reduced to three as typified by Figures 1, 2, and 3, and this will be discussed. Multiplying together the numbers involved in the above groups yields a product of 144, indicating that there are 144 facets to the problem to be borne in mind while studying the variable considerations. These are actually reduced to 86 in number, however, by virtue of the facts that: it is only necessary to study one relay out of three for 3-phase faults; some of the relays are not affected by some of the faults; and, the system of Figure 2 is a duplicate of that of Figure 1 insofar as phase-to-phase and 3-phase faults are concerned. These 86 facets are involved in the interpretation of the effects of the variable considerations.

Variable Considerations

The impedances of the power system are conveniently broken down into two single equivalent impedances for each sequence network, which are the impedance of the source up to the relay location, and the impedance of the transmission line from the relay to the point of fault. The impedance of the line may vary from zero representing a close in solid fault, to a relatively large value representing either a fault far removed from the relay location, or else a fault involving an arc of very considerable length. For any given line impedance to the point of fault, the source impedance may be relatively small, large, or in between. The impedance angle of the line may be different from that of the source. The author has considered that the line angle may vary from zero degrees (arcing fault) to 90 degrees lagging, and that the source impedance angle may vary between the limits of 60 to 90 degrees lagging. The zero sequence impedances for the source and line may differ in relative magnitude and impedance angle with respect to each other and quite independently of the positive and negative sequence network impedances. The 86 facets of the problem are interpreted in the light of these variable considerations.

There are other variables which are not covered in this study. These are concerned with the effects of load current and synchronizing current during faults, and the variations possible between the positive and negative sequence networks. This study is limited to the effects of short circuit current only, and is based on the assumption that the positive and negative sequence impedances are identical. In certain special cases, it may be necessary to consider the effect of load current, but it is believed that such cases will be relatively infrequent.

The Four Commonly Used Connections

Directional element connections are conveniently and popularly described in terms of the angle by which unity power factor balanced load current flowing in the tripping direction leads the voltage applied to the relay potential coil with due

Paper 50-233, recommended by the AIEE Relay Committee and approved by the AIEE Technical Program Committee for presentation at the AIEE Fall General Meeting, Oklahoma City, Okla., October 23-27, 1950. Manuscript submitted July 26, 1950; made available for printing August 23, 1950.

W. K. SONNEMANN is with the Westinghouse Electric Corporation, Newark, New Jersey.

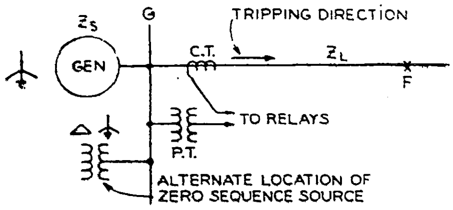


Figure 1. System diagram showing relay location and system constants

consideration given to the polarity of the relay coils. In order that no confusion will exist with regard to what is meant by the terminology used in describing the four commonly used connections, Table IV identifies these connections in terms of the voltage and current used on each relay. In this table, as well as throughout this paper, the double subscript voltage notation used is in accordance with the concept recently recommended to the AIEE Standards Committee for consideration as a recommended standard practice, the sense of which is that when the vector quantity so represented is to be considered as being in the positive half cycle, there is a drop in potential from the terminal of the first subscript to the terminal of the second subscript. Similarly, the single subscript notation used with the currents identify them by phases and in the sense that the vector quantities (currents) so represented are to be considered positive when actually flowing through the line in the tripping direction.

Method of Approach

In the great majority of cases, a sufficient answer is obtained if it is determined whether the torque on the directional element is contact opening or contact closing. This might be termed a qualitative analysis, and is determined purely by the relative phase angle between the current and the voltage applied to the relay and a consideration of the phase-angle characteristics of the relay. In a quantitative analysis, it would be necessary to multiply together the magnitudes of the voltage and current and some function of the phase angle between them as modified by the angle by which the maximum torque line of the relay leads the reference voltage vector, thus determining the magnitude of the torque as well as its direction. The qualitative analysis method serves the purpose, remembering that if the voltage is zero, the directional element will not operate anyway, and this method greatly simplifies the problem. The question then becomes that of determining the relative phase angle between the current and voltage applied to the various relays for the various connections for the various faults. To illustrate the method used, consider relay A operating with the 90-degree connection. It is actuated by the voltage V_{bc} and the current I_a. We may arbitrarily write

$$Z_D = \frac{V_{bc}}{I_a} \qquad (1)$$

in which Z_D represents an artificial impedance value obtained by dividing V_{bc} by I_a. Recalling the principles of vector algebra, it is remembered that when the operation is done in polar form, the characteristic angle of the quotient (Z_D) is obtained by subtracting the angle of the denominator (I_a) from the angle of the numerator (V_{bc}). Using the mathematicians' concept for positive angle, it is clear that when this is done, the resulting angle characteristic of Z_D will be that angle by which V_{bc} leads I_a, or, conversely, the angle by which I_a lags V_{bc}. This is exactly the kind of answer that is wanted to determine directional element operation.

It was found through making a general analysis by the method of symmetrical components that the relay voltage and relay current could be reduced to terms involving only the source voltage and the system impedance constants such that when one is divided by the other, the source voltage cancels out and the remaining expression consists of one or more system impedance values together with complex operators. With the expression for Z_D thus being reduced to a function of impedance, it is possible to substitute the appropriate values for the impedances involved for any particular system location and determine the characteristic angle for Z_D for that particular condition, and thereby determine the angle by which the relay current lags the relay voltage. The work of deriving the Z_D formulas for the 86 facets of the problem is given in the Appendixes. All quantities are kept in primary terms, neglecting the instrument transformer ratios.

Tabulation of Results

Tables I, II, and III show the final form of the Z_D formulas. These formulas are applicable to any system which can be reduced to a simplified equivalent represented by Figures 1, 2, or 3, respectively. These three figures are believed to be adequate to represent the portion of interest of any system. While it is possible to make specific applications of the formulas to fit particular conditions, it is desirable to study the formulas in general

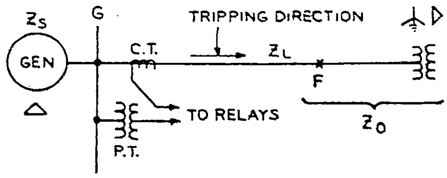


Figure 2. System with zero sequence source concentrated at opposite end of transmission line

terms on the basis of determining the maximum possible spread of possibilities. This has been done and the results are shown in Tables V, VI, and VII, corresponding respectively to Tables I, II, and III, and Figures 1, 2, and 3. In Tables V, VI, and VII the relay voltage vector is shown as the reference vector in each case, extending along the x-axis, and the maximum spread of angular position of the current with respect to it, either leading or lagging or both, is shown by the shaded area and with symbolic notation that will be explained.

Systems Studied

Figure 1 represents the simplest system layout wherein the relays at bus G protect the line from G to the fault at F. In general terms, Z_L represents the impedance of the line to the point of fault and includes the arc resistance, if any. The source is represented schematically by the generator having an impedance Z_S. The positive and negative sequence source circuit may or may not include the zero sequence source circuit, so that an alternate location is shown for the latter, illustrating the freedom of variation of sequence source impedances. This figure is adaptable to a section of any system not otherwise bound to be considered in line with Figures 2 or 3. For example, a system section (not shown) feeding the fault from the right-hand side and having similar sequence current distribution factors will not affect the section shown other than through increasing the apparent magnitude of the arc resistance. This contingency is taken care of by the freedom of variation of the line impedance Z_L in the formulas. Again, other ties be-

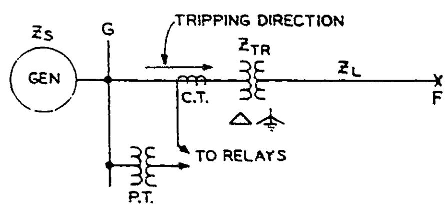


Figure 3. System with transformer bank located between the relay and the fault

Table I. System Figure 1—Z_D Values

CONNECTIONS	PHASES FAULTED	RELAY A	RELAY B	RELAY C
90°	BC	∞	$\sqrt{3}Z_{1S}\angle 270^\circ + 2Z_{1L}\angle 240^\circ$	$\sqrt{3}Z_{1S}\angle 270^\circ + 2Z_{1L}\angle 300^\circ$
	A-G	$\frac{1}{\sqrt{3}}\left[2Z_{1S}+Z_{0S}+2Z_{1L}+Z_{0L}\right]\angle 270^\circ$	∞	∞
	BC-G	∞	$\frac{\sqrt{3}AZ_{1S}\angle 330^\circ+(1+2A)Z_{1L}\angle 300^\circ}{1+A\angle 60^\circ}$	$\frac{\sqrt{3}AZ_{1S}\angle 210^\circ+(1+2A)Z_{1L}\angle 240^\circ}{1+A\angle 300^\circ}$
	ABC	$\sqrt{3}Z_{1L}\angle 270^\circ$		
60°*1	BC	$\sqrt{3}Z_{1S}\angle 270^\circ + 2Z_{1L}\angle 240^\circ$	$\frac{\sqrt{3}}{2}Z_{1S}\angle 270^\circ + Z_{1L}\angle 300^\circ$	$2Z_{1L}$
	A-G	$\frac{1}{\sqrt{3}}\left[Z_{1S}\angle 270^\circ+(Z_{0S}+2Z_{1L}+Z_{0L})\angle 330^\circ\right]$	∞	$\frac{1}{\sqrt{3}}\left[2Z_{1S}+Z_{0S}+2Z_{1L}+Z_{0L}\right]\angle 270^\circ$
	BC-G	$\frac{\sqrt{3}AZ_{1S}\angle 330^\circ+(1+2A)Z_{1L}\angle 300^\circ}{1+A\angle 60^\circ}$	$\frac{\sqrt{3}AZ_{1S}\angle 270^\circ+(1+2A)Z_{1L}\angle 300^\circ}{1+2A}$	$\frac{(1+2A)Z_{1L}\angle 300^\circ}{1+A\angle 300^\circ}$
	ABC	$Z_{1L}\angle 300^\circ$		
60°*2	BC	∞	$\frac{2}{\sqrt{3}}\left[Z_{1S}+Z_{1L}\right]\angle 270^\circ$	$\frac{1}{\sqrt{3}}\left[Z_{1S}\angle 270^\circ+2Z_{1L}\angle 330^\circ\right]$
	A-G	$\frac{1}{\sqrt{3}}\left[Z_{1S}\angle 270^\circ+Z_{0S}\angle 330^\circ\right]+\frac{1}{3}\left[2Z_{1L}+Z_{0L}\right]\angle 300^\circ$	∞	∞
	BC-G	∞	$\frac{1}{\sqrt{3}}\left[\frac{2AZ_{1S}+Z_{0S}+(1+2A)Z_{1L}}{1+A\angle 60^\circ}\right]\angle 330^\circ$	$\frac{1}{\sqrt{3}}\left[\frac{AZ_{1S}\angle 210^\circ+Z_{0S}\angle 30^\circ+(1+2A)Z_{1L}\angle 270^\circ}{1+A\angle 300^\circ}\right.$
	ABC	$Z_{1L}\angle 300^\circ$		
30°	BC	∞	$\sqrt{3}Z_{1S}\angle 270^\circ + 2Z_{1L}\angle 300^\circ$	$2Z_{1L}$
	A-G	$\frac{1}{\sqrt{3}}\left[Z_{1S}\angle 270^\circ+(Z_{0S}+2Z_{1L}+Z_{0L})\angle 330^\circ\right]$	∞	∞
	BC-G	∞	$\frac{\sqrt{3}AZ_{1S}\angle 330^\circ+(1+2A)Z_{1L}}{1+A\angle 60^\circ}$	$\frac{(1+2A)Z_{1L}\angle 300^\circ}{1+A\angle 300^\circ}$
	ABC	$\sqrt{3}Z_{1L}\angle 330^\circ$		

tween a system section to the right of the fault and the bus G have the effect of changing the value of the source impedance Z_S to use in the formulas as the fault is moved down the line.

There is one effect of a source circuit to the right of the fault which is not adequately represented in Figure 1 and the formulas pertaining thereto, as follows. With a relatively low zero sequence impedance in the grounding source connected to bus G, there tends to be a preponderance of zero sequence current in the line compared to positive and negative sequence currents as faults involving ground are moved closer to the bus G,

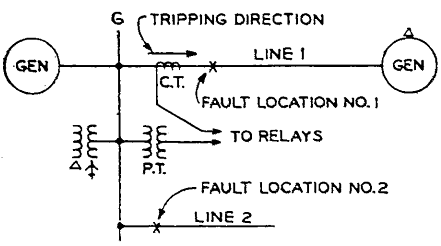


Figure 4. System showing alternate fault locations

and to a degree depending upon the magnitude of the positive and negative sequence source impedances to the right of the fault compared with those to the left of the relay. This is illustrated in more detail in Figure 4, fault location 1. With a preponderance of zero sequence current from the left, the effect will be to produce fault current in B and C phases for a phase-A-to-ground fault, and in A phase for a phase-B-to-C-to-ground fault. The formulas in this paper do not take care of this situation. It is dismissed from analysis in this paper in view of the fact that such an analysis does not appear to be strictly necessary for the following reasons:

First, it is the prime function of a protective relay to operate when the fault lies in its protected section. Single-phase-to-ground faults are to be cleared by ground relays and they may be eliminated from further consideration because this paper is concerned with directional elements for phase relays. Two-phase-to-ground faults, however, do lie within the domain of phase relays but generally more so when the system is grounded through an impedance. In this illustration the zero sequence impedance connected to bus G is low, no additional impedance being used in the neutral, or else the effect under discussion would exist only to a limited extent. Under these conditions, it may be properly expected that for the more aggravated conditions, even the 2-phase-to-ground fault may or should be properly cleared by the ground relay because of the magnitude of the zero sequence current. With all of this shifting the responsibility to the ground relay, however, it is believed as a result of a separate vector analysis not included here that the preponderance of zero sequence current for a close in fault involving ground would not prevent the correct phase relay directional element from operating.

Secondly, it is a function of protective relays not to trip when the fault is in some other location. If one of the 3-phase relays gives an incorrect directional indication for fault location 1 in Figure 4, it is of no consequence as long as tripping is obtained by another relay. The question then arises if the incorrect

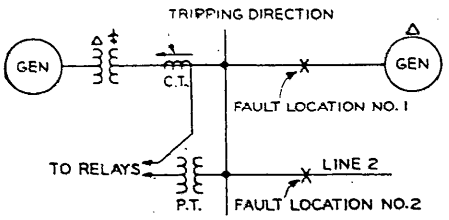


Figure 5. System diagram showing relay location requiring special consideration

indication can cause false tripping if the fault is back of the relay, as in fault location 2. There are two answers to this question. The answer for the relay location as shown in Figure 4 is that the conditions affecting the relays at bus *G* on line 1 are completely changed by a fault at location 2 on line 2. The preponderance of zero sequence current disappears from all parts of line 1, and the analysis of relay action for this case is made by considering the relays of Figure 2 in reverse.

The second answer is involved when the relay location is as shown in Figure 5, and connected for the tripping direction as shown. In this case, with fault location 1 or 2, and with a heavy preponderance of zero sequence current from the left, it appears quite likely that false tripping might result with three of the commonly used connections, which are the 90 degree, the 60 degree number 2, and the 30 degree. This is because of the tendency of the large zero sequence components to draw the three line currents together in phase position, and since these are used with three different voltages in the three different elements, it appears that at least one is likely to operate incorrectly. If the connection identified as 60 degree number 1 is used, however, the difficulties are removed inasmuch as the relay is then not sensitive to zero sequence components, either current or voltage.

Figure 2 represents the opposite condition to that discussed for Figures 1, 4, and 5 in that all of the zero sequence comes from the far end of the line so that the relays are operable from positive and negative sequence currents only. Since the result of this study can be applied in reverse to cover fault location 2 in Figure 4, it is important that all directional elements associated with fault detecting elements (such as overcurrent elements, impedance elements, and so forth) receiving enough current to operate should give a correct indication. The study of Figure 2 does not repeat the consideration of phase-to-phase and 3-phase faults, as in this respect, Figure 2 is a duplicate of Figure 1.

Figure 3 represents a typical condition wherein a delta star connected transformer bank is interposed between the relay location and the fault. Here again, it is important that all directional elements associated with fault detecting elements receiving enough current to operate should give a correct indication inasmuch as the conditions affecting the relays of Figure 3 may be applied in reverse to other relays at bus *G* protecting source circuits feeding into bus *G*.

The study of Figure 3 has been based on the connections as shown schematically in that the star side of the bank is connected to the line. Further, the line voltages *X*, *Y*, *Z*, lead the bus voltages *A*, *B*, *C* by 30 degrees in accordance with American Standards Association standard practice based on the line having the higher voltage. Other combinations were

Table II. System Figure 2—Z_D Values

CONNECTIONS	PHASES FAULTED	RELAY A	RELAY B	RELAY C
90°	A-G BC-G	$\frac{\sqrt{3}}{2} Z_T \angle 270^\circ$ $\sqrt{3}(1+2A) Z_{1L} \angle 270^\circ$	$\sqrt{3}\left[Z_{1S}\angle 270^\circ+(Z_0+2Z_{1L})\angle 330^\circ\right]$ $\frac{3AZ_{1S}\ 300^\circ+\sqrt{3}(1+2A)Z_{1L}\angle 270^\circ}{1+\sqrt{3}A\angle 30^\circ}$	$\sqrt{3}\left[Z_{1S}\angle 270^\circ+(Z_0+2Z_{1L})\angle 210^\circ\right]$ $\frac{3AZ_{1S}\angle 240^\circ+\sqrt{3}(1+2A)Z_{1L}\angle 270^\circ}{1+\sqrt{3}A\angle 330^\circ}$
60° #1	A G BC-G	$\frac{1}{\sqrt{3}}\left[Z_{1S}\angle 270^\circ+(Z_0+2Z_{1L})\angle 330^\circ\right]$ $\frac{\sqrt{3}AZ_{1S}\angle 330^\circ+(1+2A)Z_{1L}\angle 300^\circ}{1+A\angle 60^\circ}$	∞ $\frac{\sqrt{3}AZ_{1S}\angle 270^\circ+(1+2A)Z_{1L}\angle 300^\circ}{1+2A}$	$\frac{1}{\sqrt{3}} Z_T \angle 270^\circ$ $\frac{(1+2A)Z_{1L}\angle 300^\circ}{1+A\angle 300^\circ}$
60° #2	A-G BC-G	$\frac{1}{2}\left[\sqrt{3}Z_{1S}\angle 270^\circ+\sqrt{3}Z_0\angle 330^\circ+2Z_{1L}\angle 300^\circ\right]$ $(1+\sqrt{3}A\angle 330^\circ)Z_{1L}\angle 300^\circ$	$2Z_{1L}$ $\frac{\sqrt{3}AZ_{1S}\angle 300^\circ+(1+3A)Z_{1L}\angle 300^\circ}{1+\sqrt{3}A\angle 30^\circ}$	$\sqrt{3}Z_{1S}\angle 270^\circ+\sqrt{3}Z_0\angle 210^\circ+2Z_{1L}\angle 240^\circ$ $\frac{(1+\sqrt{3}A\angle 30^\circ)Z_{1L}\angle 300^\circ}{1+\sqrt{3}A\angle 330^\circ}$
30°	A-G BC-G	$\frac{\sqrt{3}}{2}\left[Z_{1S}\angle 270^\circ+(Z_0+2Z_{1L})\angle 330^\circ\right]$ $3AZ_{1S}+\sqrt{3}(1+2A)Z_{1L}\angle 330^\circ$	$\sqrt{3}\left[Z_{1S}\angle 90^\circ+(Z_0+2Z_{1L})\angle 30^\circ\right]$ $\frac{3AZ_{1S}\angle 300^\circ+\sqrt{3}(1+2A)Z_{1L}\angle 330^\circ}{1+\sqrt{3}A\angle 30^\circ}$	$\sqrt{3}Z_T\angle 270^\circ$ $\frac{\sqrt{3}(1+2A)Z_{1L}\angle 330^\circ}{1+\sqrt{3}A\angle 330^\circ}$

Table III. System Figure 3—Z_D Values

Connections	Phases Faulted	Relay A	Relay B	Relay C
90°	YZ	$\sqrt{3}[Z_{1S}\angle 270^\circ+2(Z_{1TR}+Z_{1L})\angle 330^\circ]$	$\sqrt{3}[Z_{1S}\angle 270^\circ+2(Z_{1TR}+Z_{1L})\angle 210^\circ]$	$\sqrt{3}[Z_{1S}+(Z_{1TR}+Z_{1L})]\angle 270^\circ$
	X-G	$\sqrt{3}Z_{1S}\angle 270^\circ+[2Z_{1TR}+2Z_{1L}+Z_0]\angle 240^\circ$	$\sqrt{3}Z_{1S}\angle 270^\circ+[2Z_{1TR}+2Z_{1L}+Z_0]\angle 300^\circ$	∞
	YZ-G	$\frac{\sqrt{3}[AZ_{1S}\angle 210^\circ+(1+2A)(Z_{1TR}+Z_{1L}\angle 270^\circ]}{1+A\angle 300^\circ}$	$\frac{\sqrt{3}[AZ_{1S}\angle 330^\circ+(1+2A)(Z_{1TR}+Z_{1L})\angle 270^\circ]}{1+A\angle 60^\circ}$	$\frac{\sqrt{3}[2AZ_{1S}+(1+2A)(Z_{1TR}+Z_{1L})]\angle 270^\circ}{1+2A}$
	XYZ	$\sqrt{3}(Z_{1TR}+Z_{1L})\angle 270^\circ$		
60°*1	YZ	∞	$\frac{2}{\sqrt{3}}[Z_{1S}+(Z_{1TR}+Z_{1L})]\angle 270^\circ$	$\frac{1}{\sqrt{3}}[Z_{1S}\angle 270^\circ+2(Z_{1TR}+Z_{1L})\angle 330^\circ]$
	X-G	$\frac{1}{2}[\sqrt{3}Z_{1S}\angle 270^\circ+(2Z_{1TR}+2Z_{1L}+Z_0)\angle 300^\circ]$	$2Z_{1TR}+2Z_{1L}+Z_0$	$\sqrt{3}Z_{1S}\angle 270^\circ+[2Z_{1TR}+2Z_{1L}+Z_0]\angle 240^\circ$
	YZ-G	$AZ_{1S}+(1+2A)(Z_{1TR}+Z_{1L})\angle 300^\circ$	$\frac{[2AZ_{1S}+(1+2A)(Z_{1TR}+Z_{1L})]\angle 300^\circ}{1+\sqrt{3}A\angle 30^\circ}$	$\frac{AZ_{1S}\angle 240^\circ+(1+2A)(Z_{1TR}+Z_{1L})\angle 300^\circ}{1+\sqrt{3}A\angle 330^\circ}$
	XYZ	$(Z_{1TR}+Z_{1L})\angle 300^\circ$		
60°*2	YZ	$2(Z_{1TR}+Z_{1L})$	$\sqrt{3}Z_{1S}\angle 270^\circ+2(Z_{1TR}+Z_{1L})\angle 240^\circ$	$\frac{\sqrt{3}}{2}Z_{1S}\angle 270^\circ+(Z_{1TR}+Z_{1L})\angle 300^\circ$
	X-G	$\frac{2}{\sqrt{3}}[Z_{1S}+Z_{1TR}+Z_{1L}+\frac{1}{2}Z_0]\angle 270^\circ$	$\frac{1}{\sqrt{3}}[Z_{1S}\angle 270^\circ+(2Z_{1TR}+2Z_{1L}+Z_0)\angle 330^\circ]$	∞
	YZ-G	$\frac{(1+2A)(Z_{1TR}+Z_{1L})\angle 300^\circ}{1+A\angle 300^\circ}$	$\frac{\sqrt{3}AZ_{1S}\angle 330^\circ+(1+2A)(Z_{1TR}+Z_{1L})\angle 300^\circ}{1+A\angle 60^\circ}$	$\frac{\sqrt{3}AZ_{1S}\angle 270^\circ+(1+2A)(Z_{1TR}+Z_{1L})\angle 300^\circ}{1+2A}$
	XYZ	$(Z_{1TR}+Z_{1L})\angle 300^\circ$		
30°	YZ	$\sqrt{3}[Z_{1S}\angle 90^\circ+2(Z_{1TR}+Z_{1L})\angle 30^\circ]$	$2\sqrt{3}[Z_{1S}+(Z_{1TR}+Z_{1L})]\angle 270^\circ$	$\sqrt{3}[Z_{1S}\angle 270^\circ+(Z_{1TR}+Z_{1L})\angle 330^\circ]$
	X-G	$\sqrt{3}Z_{1S}\angle 270^\circ+(2Z_{1TR}+2Z_{1L}+Z_0)\angle 300^\circ$	$2Z_{1TR}+2Z_{1L}+Z_0$	∞
	YZ-G	$\frac{\sqrt{3}[AZ_{1S}\angle 30^\circ+(1+2A)(Z_{1TR}+Z_{1L})\angle 330^\circ]}{1+A\angle 300^\circ}$	$\frac{\sqrt{3}[2AZ_{1S}\angle 330^\circ+(1+2A)(Z_{1TR}+Z_{1L})\angle 330^\circ]}{1+A\angle 60^\circ}$	$\frac{\sqrt{3}[AZ_{1S}\angle 270^\circ+(1+2A)(Z_{1TR}+Z_{1L})\angle 330^\circ]}{1+2A}$
	XYZ	$\sqrt{3}(Z_{1TR}+Z_{1L})\angle 330^\circ$		

considered, such as star (low-voltage) to delta (high-voltage), and with the nonstandard condition of the 30-degree lagging relationship. It was found that the same results were obtained, the only difference being that when the 30 degree lagging connections were used, the formulas became associated with relays for different phases. Hence it follows that what is the best connection for Figure 3 also is the best for the other possible transformer connections.

The Search for Limits

With the formulas derived as shown in Tables I, II, and III it became desirable to analyze these formulas to determine the maximum spread in phase angle between the current and the voltage. The results of this study are shown in Tables V, VI, and VII. Several of the vector diagrams in these tables will now be discussed in order to clarify the method of analysis used as well as the peculiar notation used with the diagrams. Dash lines in the table mean that that particular relay received no current for the fault in question. Blank spaces in the *B* and *C* relay columns for the 3-phase fault, *ABC*, indicate that the consideration given one relay, relay *A*, is sufficient.

Remembering that Table V is associated with Table I and Figure 1, it is determined that the diagram for relay *B*, line 1 (90-degree connection, phase-*B*-to-*C* fault) comes from the formula

$$Z_D=\sqrt{3}Z_{1S}\angle 270^\circ+2Z_{1L}\angle 240^\circ \quad (2)$$

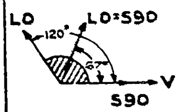
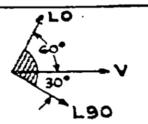
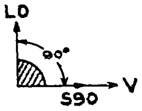
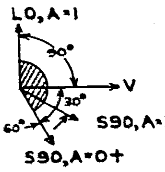
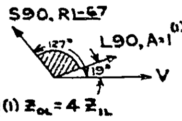
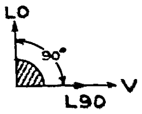
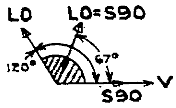
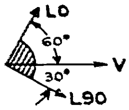
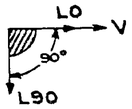
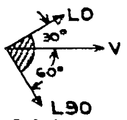
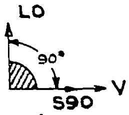
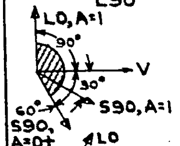
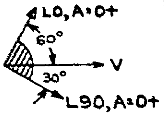
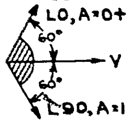
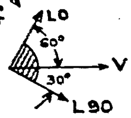
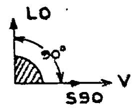
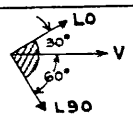
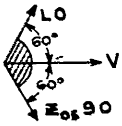
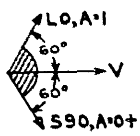
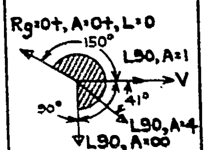
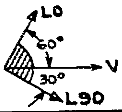
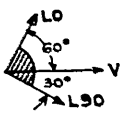
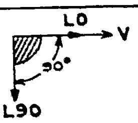
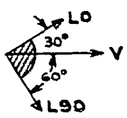
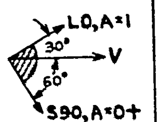
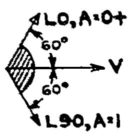
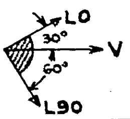
The angle of Z_D is the angle by which the current lags the voltage, so that the current vector will be plotted clockwise from the reference voltage vector by whatever positive angle results from evaluating equation 2. The impedance terms Z_{1S} and Z_{1L} are the positive sequence impedances of the source and the line, respectively. The expressions $\angle 270^\circ$ and $\angle 240^\circ$ are mathematical operators producing a positive angular rotation of the amount indicated. In this particular formula, $-j$ could have been used for $\angle 270^\circ$ and the a^2 operator of symmetrical components could have been used for $\angle 240^\circ$. These two substitutions were not made, however, because so many of the other formulas contain operators in steps of 30 degrees that is was considered preferable to keep all of the operators in this form.

Equation 2 is to be studied for limits. One limit is apparent immediately in that if the fault is close enough, Z_{1L} becomes zero and all that is left is $\sqrt{3}Z_{1S}\angle 270^\circ$.

Table IV. Table of Connections

Connections	Relay A		Relay B		Relay C	
	Voltage	Current	Voltage	Current	Voltage	Current
90°	V_{bc}	I_a	V_{ca}	I_b	V_{ab}	I_c
60° No. 1	V_{ac}	I_a-I_b	V_{ba}	I_b-I_c	V_{cb}	I_c-I_a
60° No. 2	$-V_{cn}$	I_a	$-V_{an}$	I_b	$-V_{bn}$	I_c
30°	V_{ac}	I_a	V_{ba}	I_b	V_{cb}	I_c

Table V. System Figure 1

CONNECTIONS	PHASES FAULTED	RELAY A	RELAY B	RELAY C
90°	BC	—		
	AG		—	—
	BC-G	—		
	ABC			
60° #1	BC			
	AG		—	
	BC-G			
	ABC			
60° #2	BC	—		
	A-G		—	—
	BC-G	—		
	ABC			
30°	BC	—		
	A-G		—	—
	BC-G	—		
	ABC			

The source cannot have an angle greater than 90 degrees, so if we take 90 degrees for the angle of the source impedance and rotate it by $\underline{/270°}$, the answer comes out at an angle of 360 or zero degrees. This is the characteristic angle of Z_D and thus means that the current and voltage are in phase. Referring to the vector diagram in Table V, it will be seen that a current vector has been drawn in phase with the voltage vector, and the notation for it is *S*90. This notation means that the source impedance (*S* for source) governs for this limit, and that it has an angle of 90 degrees.

Assume that the source impedance has an angle of 60 degrees. Adding 60 to 270 degrees yields 330 degrees, meaning that the current lags the voltage by 330 degrees (leads by 30 degrees) and such a current vector would fall in the shaded area, not establishing a limit, and hence was not plotted.

For this case, equation 2, the maximum possible angular position was found by assuming that the fault is close in and involves an arc of great length so that the impedance of the line is not only at 0 degrees, but also so large that the impedance of the source becomes negligible. In this case, through neglecting Z_{1S}, the expression for Z_D is evaluated by considering the second term of equation 2, or $2Z_{1L}\underline{/240°}$, and with Z_{1L} having its own angle of zero degrees, it follows that the current leads the voltage by 120 degrees.

Another current vector is shown accordingly in the vector diagram, and wherein the notation *LO* signifies that the line impedance governs and that it has an angle of zero degrees. It was felt that it was a bit pessimistic to lean too heavily on the limit thus established because this calls for the arcing fault to absorb the entire system voltage, even though arcs on horizontally spaced lines approaching full system voltage have been reported. Consequently, a compromise condition was evaluated as shown by the current vector at 67 degrees leading, wherein the notation *LO*=*S*90 means that the line impedance at zero degrees (all arc) was taken numerically equal to the source impedance at 90 degrees and Z_D evaluated accordingly, giving due attention to the $\sqrt{3}$ and 2 factors. This would account for the arc absorbing somewhat more than half of the available voltage. Other intermediate vectors representing alternate conditions are occasionally used in other diagrams throughout these tables.

The method of analyzing the formulas is thus established as a study to determine what possible impedance values, both in angle and in magnitude, can be utilized to yield the maximum angle and the minimum angle in either direction, and anything of a more reasonable nature as a limit in between.

As might be expected, the 2-phase-to-ground fault condition was more difficult to handle, both in derivation and analysis, than the other three combined. The only practical way which could be found was to introduce a factor A, a real or complex number, letting

$$A=\frac{Z_0}{Z_1} \qquad (3)$$

in which Z_0 and Z_1 represents the final equivalent single impedances for the entire zero and positive sequence networks, respectively, as used in calculating the fault current components. The factor A is thus a ratio, and may vary from slightly more than zero to a very large number. In general, however, its possibilities were considered on the basis of it being either 0+ or 1. Large values of A make the 2-phase-to-ground fault approach the conditions of a phase-to-phase fault, as may be verified by comparing the formulas. (The one case where this does not hold true is for the 60°, number 2 connection, Figures 1 and 2, which received the necessary special attention.)

Typical of a case where A is taken at 0+, consider Relay B, line 3, Table V. The formula is

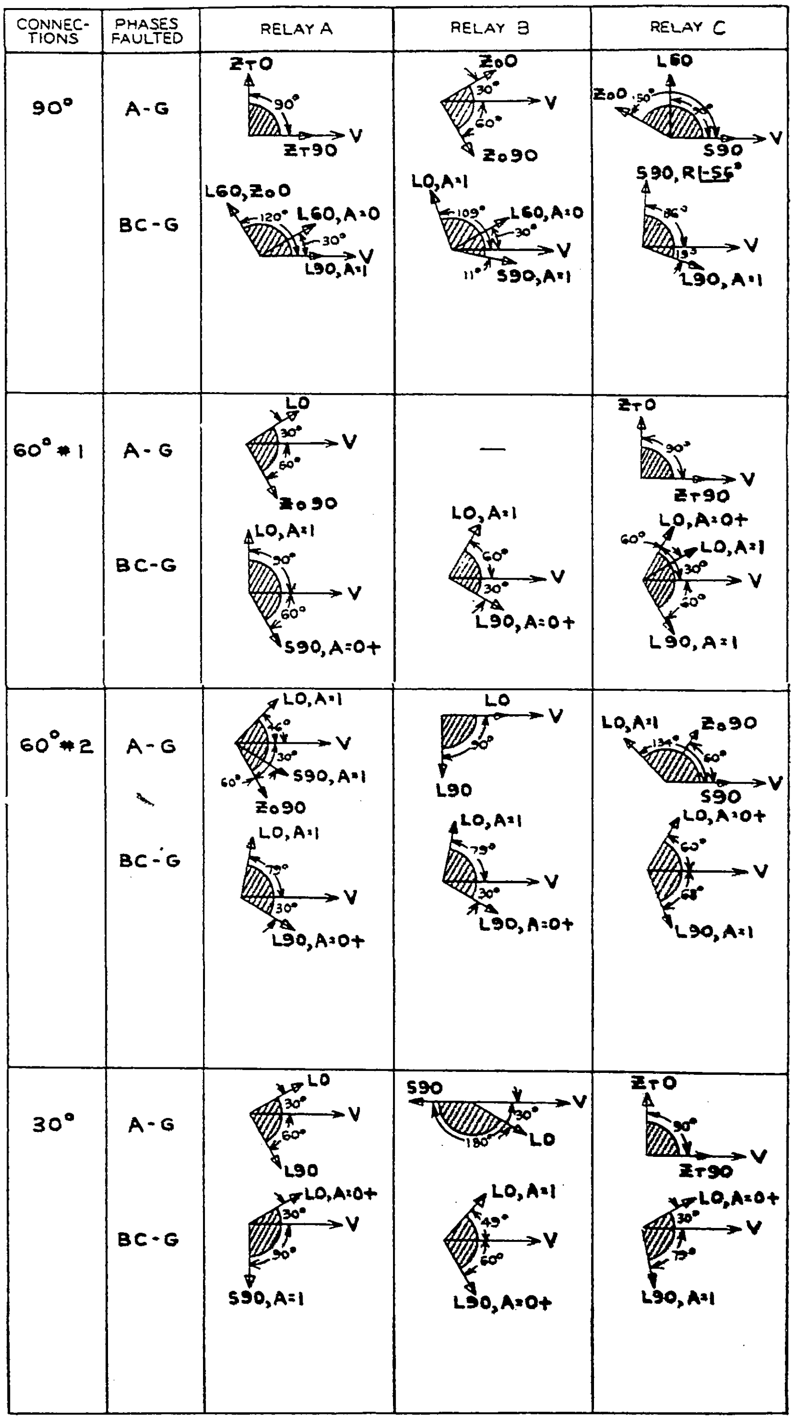
$$Z_D=\frac{\sqrt{3}AZ_{1S}\underline{/330^\circ}+(1+2A)Z_{1L}\underline{/300^\circ}}{1+A\underline{/60^\circ}} \qquad (4)$$

Consider the limiting condition when the line impedance is zero so that the second term of the numerator drops out. Now assume that A is very small, and a real number (meaning that Z_0 and Z_1 have the same angle). The denominator is then practically unity and may be neglected. This leaves the angle to be determined by $Z_{1S}\underline{/330^\circ}$, and with a 90-degree source circuit, this yields 60 degrees for current lagging voltage. A current vector is accordingly shown at this angle in the vector diagram with the notation $S90$, $A=0+$.

It is of importance of note that the magnitude of Z_D, heretofore not discussed, is a measure of the ratio of the voltage to the current. If this ratio becomes low enough, the relay will not operate. The condition just discussed approaches this condition, where $A=0+$. An auxiliary condition was calculated for $A=1$, a real number, yielding an angle of 30 degrees lagging, which has been shown.

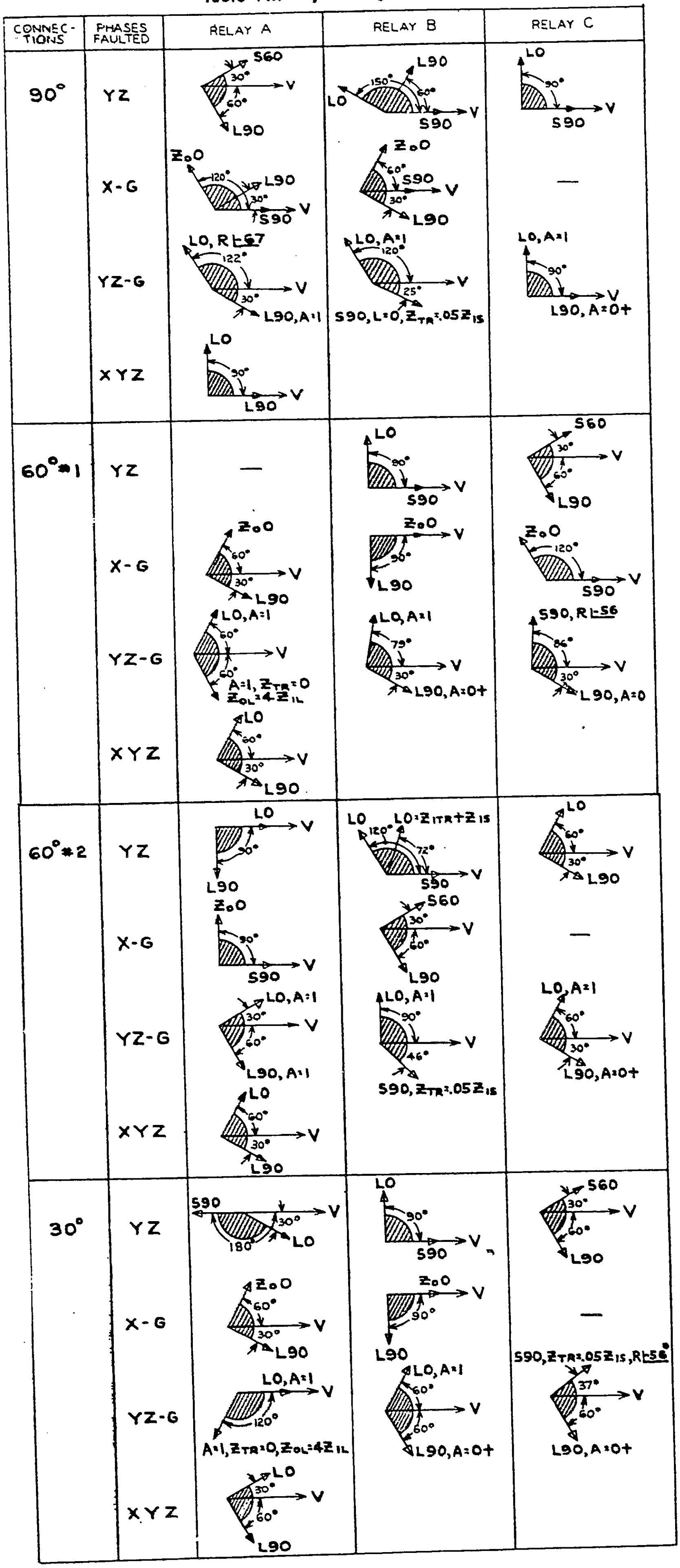
A value of $A=1$ can be obtained three ways. First, at zero line impedance, the ratio of source impedances can be unity. Secondly, if the line impedance is represented by a high value of arc resistance, A automatically becomes one. Such a condition forms the 90-degree current leading limit for the diagram under discussion. Thirdly, an assumption has been made that, for the sake of even figures, the zero sequence impedance of the line is equal to four times the positive sequence impedance, hence if the fault is appropriately located a sufficient distance down the line, A becomes unity, even with a low zero sequence source impedance.

Table VI. System Figure 2



Transferring attention to the diagram for Relay C, line 3, Table V, a new symbol $R\underline{/-67^\circ}$ is discovered. It represents the maximum angular rotating effect of −67 degrees which could be discovered by trial calculations for the portion $A/1+A\underline{/300^\circ}$ of the formula for this relay as applied to the Z_{1S} term with $Z_{1L}=0$ and in which A is a complex number occasioned by the existence of a resistance to ground in the fault affecting the zero sequence network only, as illustrated by R_g in Figure 6. The R factor appears again at different points in these tables representing the same type of operation with

Table VII. System Figure 3



angular values appropriate to the occasion. In all cases where the R factor was used, it was evaluated on the basis that the source impedances were at angles of 90 degrees lag and that $Z_{0S}=0.05Z_{1S}$. In the same vector diagram (Relay C, line 3, Table V) the factor A is used equal to 1 based on $Z_{0L}=4Z_{1L}$.

The 2-phase-to-ground fault condition is particularly tricky to handle for the 60°, number 2 connection by virtue of the fact that the fault condition in itself is complicated enough, and with the relay using phase-to-ground voltage for polarization, the analysis becomes tedious. In this case, the R_g value Figure 6 is likely to become quite important, as may be seen by the 150-degree angle shown for Relay C, Table V, line 11. This diagram also shows a case where it was desirable to consider values for A of 1, 4, and infinity.

Other symbolic notation used in these tables is explained as follows:

$L=0$ means that the line has no impedance, contrasting with the symbol *LO* described previously.

Z_{0S} denotes the zero sequence impedance in the source up to the relay, while Z_0 denotes the total zero sequence network.

Z_T represents the total of all impedances used in making the initial fault current calculation, as for example, in calculating a single-phase-to-ground fault $Z_T=Z_1+Z_2+Z_0$.

Z_{TR} represents the impedance of the transformer in Figure 3.

In preparing Table VII for Figure 3, it was necessary in some cases to assume a very low limit for the impedance of the transformer, but not zero. In such cases, it was assumed equal to 5 per cent of the positive sequence source impedance and is so indicated.

Summary of Results

The vector diagrams now may be studied in the light of directional element phase angle characteristics either as they are, or as they should be.

Considering the 90-degree connection first, it will be recalled that when this connection is used, the relay is usually equipped with phase shifting means to put the zero torque line at 45 degrees lagging—135 degrees leading, and the directional element therefore has maximum torque when the current leads the voltage by 45 degrees. A study of the vector diagrams reveals three cases where the current falls in the contact opening zone by being either more than 45 degrees lagging or more than 135 degrees leading. These

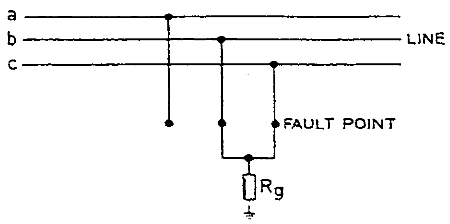


Figure 6. Location of fault resistance affecting zero sequence network only

are (1) Relay B, BC-G fault, System Figure 1; (2) Relay C, A-G fault, System Figure 2; and (3) Relay B, YZ fault, System Figure 3. The last two cases may be dismissed as nontroublesome on the basis that the conditions assumed for these limits are associated with such low current values that the fault detecting element can hardly be expected to operate. In case of doubt, the formulas may be used to investigate particular conditions short of the limits shown here. In the first case, however, a hazard of possible false operation does exist when the relay location is as shown in Figure 5 and with the fault back of the relay as in location 1 or 2. It will be remembered that for this location, a previous discussion gave the 60-degree, number 1 connection a preference for other reasons.

The 60-degree, number 1 connection is generally used with an element having a watt-type characteristic, which means that the zero torque line falls at 90 degrees leading and lagging the reference voltage, with a maximum contact closing torque produced when the current and voltage are in phase. With this type of characteristic, the only possibilities of trouble are for Relay A, BC fault, System Figure 1, and for Relay C, X-G fault, System Figure 3. In each of these cases it is doubtful if the fault detecting element will receive enough current to operate.

In the case of the 60-degree, number 2 connection, if a watt-type element is used, the first possibility of trouble is encountered in connection with System Figure 1, BC-G fault, Relay C, for the condition of $R_g=0+$, $A=0+$, $L=0$, where an incorrect directional indication would obtain. This fault would be cleared by relay B acting alone. The problem of the incorrect indication of relay C causing a false trip out is, once again, limited to a location as shown by Figure 5. Other cases where the current leads the voltage by more than 90 degrees are discovered for System Figure 2, A-G fault, Relay C, and System Figure 3, YZ fault, Relay B. The possibility of causing a false trip out by approaching these limits depends on the magnitude of current involved as it affects the fault detecting element.

The 30-degree connection remains free of trouble only as long as the application is described by System Figure 1. If applied as shown by Figures 2 or 3, it is easily seen that erroneous directional indications will result with probable current magnitudes within the range of operation of the fault detecting element, thus leading to false tripping.

Conclusions

1. The general practice of shifting the angle of maximum torque to approximately 45 degrees leading for the 90-degree connection appears to give the best characteristic to fit the spread of possible phase angles.
2. A watt-type element appears to be best suited to the three remaining types of connections.
3. Each connection theoretically involves possibility of trouble in certain extreme cases.
4. The 30-degree connection is not suited to systems as shown by Figures 2 and 3.

General Notes on Derivation of Formulas

The following appendixes I, II, and III, cover the calculation of the necessary fault currents and voltages for the four types of faults, following which the Z_D formulas appropriate to the several relay connections are derived for enough cases to illustrate the method. The sequence network connections for the various fault calculations are not shown as it is taken for granted that those who check this work will be familiar with the method of symmetrical components.

The following remarks, however, are appropriate.

V_a' represents the generated voltage which causes the fault current to flow.

Subscripts 1, 2, and 0 denote positive, negative, and zero sequence, respectively, as applied to current, voltage, or impedance.

Subscripts S and L denote source and line, respectively.

Impedance symbols with 1, 2, or 0 subscripts only denote the total impedance of the respective positive, negative, or zero sequence network to the point of fault.

The relay voltages are calculated for the bus G.

A is the ratio Z_0/Z_1.

For simplicity, phase-to-ground voltages at bus G are written without the second subscript, V_a instead of V_{ag}.

Appendix I. System Figure 1

$Z_1=Z_{1S}+Z_{1L}$
$Z_2=Z_1$
$Z_0=Z_{0S}+Z_{0L}$

Phase-to-Phase Fault on Phases BC

Calculate currents and voltages

$$I_{a1}=-I_{a2}=\frac{V_a'}{Z_T}=\frac{V_a'}{Z_1+Z_2} \quad (5)$$

$$I_a=I_{a1}+I_{a2}=0 \quad (6)$$

$$I_b=a^2I_{a1}+aI_{a2}=\frac{V_a'}{Z_T}(\underline{/240^\circ}+\underline{/300^\circ})=\frac{V_a'}{Z_T}\sqrt{3}\,\underline{/270^\circ} \quad (7)$$

$$I_c=aI_{a1}+a^2I_{a2}=\frac{V_a'}{Z_T}(\underline{/120^\circ}+\underline{/60^\circ})=\frac{V_a'}{Z_T}\sqrt{3}\underline{/90^\circ} \quad (8)$$

$$I_a-I_b=\frac{V_a'}{Z_T}\sqrt{3}\underline{/90^\circ} \quad (9)$$

$$I_b-I_c=\frac{V_a'}{Z_T}2\sqrt{3}\underline{/270^\circ} \quad (10)$$

$$I_c-I_a=\frac{V_a'}{Z_T}\sqrt{3}\underline{/90^\circ} \quad (11)$$

Voltages at relay location, bus G

$$V_{a1}=V_a'-I_{a1}Z_{1S}=V_a'-\frac{V_a'}{Z_T}Z_{1S}=\frac{V_a'}{Z_T}(Z_T-Z_{1S})=\frac{V_a'}{Z_T}(Z_{1S}+2Z_{1L}) \quad (12)$$

$$V_{a2}=-I_{a2}Z_{2S}=\frac{V_a'}{Z_T}Z_{1S} \quad (13)$$

$$V_a=V_{a1}+V_{a2}=\frac{V_a'}{Z_T}(2Z_{1S}+2Z_{1L}) \quad (14)$$

$$V_b=a^2V_{a1}+aV_{a2}=\frac{V_a'}{Z_T}(a^2Z_{1S}+2a^2Z_{1L}+aZ_{1S})=\frac{V_a'}{Z_T}(Z_{1S}\underline{/180^\circ}+2Z_{1L}\underline{/240^\circ}) \quad (15)$$

$$V_c=aV_{a1}+a^2V_{a2}=\frac{V_a'}{Z_T}(aZ_{1S}+2aZ_{1L}+a^2Z_{1S})=\frac{V_a'}{Z_T}(Z_{1S}\underline{/180^\circ}+2Z_{1L}\underline{/120^\circ}) \quad (16)$$

$$V_{ab}=V_a-V_b=\frac{V_a'}{Z_T}(3Z_{1S}+2\sqrt{3}Z_{1L}\underline{/30^\circ}) \quad (17)$$

$$V_{bc}=V_b-V_c=\frac{V_a'}{Z_T}(2\sqrt{3}Z_{1L}\underline{/270^\circ}) \quad (18)$$

$$V_{ca}=V_c-V_a=\frac{V_a'}{Z_T}(3Z_{1S}\underline{/180^\circ}+2\sqrt{3}Z_{1L}\underline{/150^\circ}) \quad (19)$$

Derive the Z_D formulas

90° CONNECTION

Relay A: $Z_D=\frac{V_{bc}}{I_a}=\infty$ (no current) (20)

Relay B: $Z_D=\frac{V_{ca}}{I_b}=$

$$\frac{3Z_{1S}\angle 180^\circ+2\sqrt{3}Z_{1L}\angle 150^\circ}{\sqrt{3}\angle 270^\circ}\times\frac{\angle 90^\circ}{\angle 90^\circ}$$

The term V_a'/Z_T is not shown because it appears in both numerator and denominator and therefore cancels out. The multiplication by $\angle 90^\circ/\angle 90^\circ$ is to convert the denominator to a real term.

$$Z_D=\sqrt{3}Z_{1S}\angle 270^\circ+2Z_{1L}\angle 240^\circ \quad (21)$$

Relay C: $Z_D=\frac{V_{ab}}{I_c}=$

$$\frac{3Z_{1S}+2\sqrt{3}Z_{1L}\angle 30^\circ}{\sqrt{3}\angle 90^\circ}\times\frac{\angle 270^\circ}{\angle 270^\circ}$$

$$=\sqrt{3}Z_{1S}\angle 270^\circ+2Z_{1L}\angle 300^\circ \quad (22)$$

60° Connection Number 1

Relay A: $Z_D=\frac{V_{ac}}{I_a-I_b}=\frac{-V_{ca}}{I_a-I_b}=$

$$\frac{3Z_{1S}+2\sqrt{3}Z_{1L}\angle 330^\circ}{\sqrt{3}\angle 90^\circ}\times\frac{\angle 270^\circ}{\angle 270^\circ}$$

$$=\sqrt{3}Z_{1S}\angle 270^\circ+2Z_{1L}\angle 240^\circ \quad (23)$$

Relay B: $Z_D=\frac{V_{ba}}{I_b-I_c}=\frac{-V_{ab}}{I_b-I_c}=$

$$\frac{3Z_{1S}\angle 180^\circ+2\sqrt{3}Z_{1L}\angle 210^\circ}{2\sqrt{3}\angle 270^\circ}\times\frac{\angle 90^\circ}{\angle 90^\circ}$$

$$=\frac{\sqrt{3}}{2}Z_{1S}\angle 270^\circ+2Z_{1L}\angle 300^\circ \quad (24)$$

Relay C: $Z_D=\frac{V_{cb}}{I_c-I_a}=\frac{-V_{bc}}{I_c-I_a}=$

$$\frac{2\sqrt{3}Z_{1L}\angle 90^\circ}{\sqrt{3}\angle 90^\circ}=2Z_{1L} \quad (25)$$

60° Connection Number 2

Relay A: $Z_D=\frac{-V_c}{I_a}=\infty \quad (26)$

Relay B: $Z_D=\frac{-V_a}{I_b}=$

$$\frac{2Z_{1S}\angle 180^\circ+2Z_{1L}\angle 180^\circ}{\sqrt{3}\angle 270^\circ}\times\frac{\angle 90^\circ}{\angle 90^\circ}$$

$$=\frac{2}{\sqrt{3}}(Z_{1S}\angle 270^\circ+Z_{1L}\angle 270^\circ) \quad (27)$$

Relay C: $Z_D=\frac{-V_b}{I_c}=$

$$\frac{Z_{1S}+2Z_{1L}\angle 60^\circ}{\sqrt{3}\angle 90^\circ}\times\frac{\angle 270^\circ}{\angle 270^\circ}$$

$$=\frac{1}{\sqrt{3}}(Z_{1S}\angle 270^\circ+2Z_{1L}\angle 330^\circ) \quad (28)$$

30° Connection

Relay A: $Z_D=\frac{V_{ac}}{I_a}=\infty \quad (29)$

Relay B: $Z_D=\frac{V_{ba}}{I_b}$

$$=\frac{3Z_{1S}\angle 180^\circ+2\sqrt{3}Z_{1L}\angle 210^\circ}{\sqrt{3}\angle 270^\circ}\times\frac{\angle 90^\circ}{\angle 90^\circ}$$

$$=\sqrt{3}Z_{1S}\angle 270^\circ+2Z_{1L}\angle 300^\circ \quad (30)$$

Relay C: $Z_D=\frac{V_{cb}}{I_c}=\frac{2\sqrt{3}Z_{1L}\angle 90^\circ}{\sqrt{3}\angle 90^\circ}=2Z_{1L} \quad (31)$

Phase A-to-Ground Fault

Calculate currents, and voltages at bus G

$$I_{a1}=I_{a2}=I_{a0}=\frac{V_a'}{Z_1+Z_2+Z_0}=\frac{V_a'}{Z_T} \quad (32)$$

$$I_a=3I_{a1}=3\frac{V_a'}{Z_T} \quad (33)$$

$$I_b=I_c=0 \quad (34)$$

$$I_a-I_b=3\frac{V_a'}{Z_T} \quad (35)$$

$$I_b-I_c=0 \quad (36)$$

$$I_c-I_a=3\frac{V_a'}{Z_T}\angle 180^\circ \quad (37)$$

$$V_{a1}=V_a'-I_{a1}Z_{1S}=V_a'-\frac{V_a'}{Z_T}Z_{1S}$$

$$=\frac{V_a}{Z_T}(Z_T+Z_{1S}\angle 180^\circ) \quad (38)$$

$$V_{a2}=-I_{a2}Z_{2S}=\frac{V_a'}{Z_T}Z_{1S}\angle 180^\circ \quad (39)$$

$$V_{a0}=-I_{a0}Z_{0S}=\frac{V_a'}{Z_T}Z_{0S}\angle 180^\circ \quad (40)$$

$$V_a=V_{a1}+V_{a2}+V_{a0}=$$

$$\frac{V_a'}{Z_T}(Z_T+2Z_{1S}\angle 180^\circ+Z_{0S}\angle 180^\circ) \quad (41)$$

But $Z_T=2Z_{1S}+2Z_{1L}+Z_{0S}+Z_{0L} \quad (42)$

$$V_a=\frac{V_a'}{Z_T}(2Z_{1L}+Z_{0L}). \quad (43)$$

$$V_b=a^2V_{a1}+aV_{a2}+V_{a0}$$

$$=\frac{V_a'}{Z_T}(Z_T\angle 240^\circ+Z_{1S}\angle 60^\circ+Z_{1S}\angle 300^\circ+Z_{0S}\angle 180^\circ)$$

$$=\frac{V_a'}{Z_T}(\sqrt{3}Z_{1S}\angle 270^\circ+\sqrt{3}Z_{0S}\angle 210^\circ+2Z_{1L}\angle 240^\circ+Z_{0L}\angle 240^\circ) \quad (44)$$

$$V_c=aV_{a1}+a^2V_{a2}+V_{a0}$$

$$=\frac{V_a'}{Z_T}(Z_T\angle 120^\circ+Z_{1S}\angle 300^\circ+Z_{1S}\angle 60^\circ+Z_{0S}\angle 180^\circ)$$

$$=\frac{V_a'}{Z_T}(\sqrt{3}Z_{1S}\angle 90^\circ+\sqrt{3}Z_{0S}\angle 150^\circ+2Z_{1L}\angle 120^\circ+Z_{0L}\angle 120^\circ) \quad (45)$$

The phase-to-phase voltages are derived as in a preceding section by taking the difference between the appropriate phase-to-ground voltages, above. Substitutions are made within the parenthesis for Z_T in accordance with equation 42 in order to separate the source and line impedance terms so that the end results are:

$$V_{ab}=\frac{V_a'}{Z_T}(\sqrt{3}Z_{1S}\angle 90^\circ+\sqrt{3}Z_{0S}\angle 30^\circ+2\sqrt{3}Z_{1L}\angle 30^\circ+\sqrt{3}Z_{0L}\angle 30^\circ) \quad (46)$$

$$V_{bc}=\frac{V_a'}{Z_T}(2\sqrt{3}Z_{1S}\angle 270^\circ+\sqrt{3}Z_{0S}\angle 270^\circ+2\sqrt{3}Z_{1L}\angle 270^\circ+\sqrt{3}Z_{0L}\angle 270^\circ) \quad (47)$$

$$V_{ca}=\frac{V_a'}{Z_T}(\sqrt{3}Z_{1S}\angle 90^\circ+\sqrt{3}Z_{0S}\angle 150^\circ+2\sqrt{3}Z_{1L}\angle 150^\circ+\sqrt{3}Z_{0L}\angle 150^\circ) \quad (48)$$

The relay formulas are derived in the same manner as in a preceding section. The expression for current in each case is so simple that performing the necessary division of current into voltage is done in one step, yielding at once the formulas which are already tabulated in Table I for phase-A-to-ground fault, hence there is no point in repeating them here.

Two-Phase-to-Ground Fault on Phases B and C

The sequence networks are, of course, connected in parallel, and from this consideration

$$Z_T=Z_1+\frac{Z_2Z_0}{Z_2+Z_0}$$

Letting

$$A=\frac{Z_0}{Z_1} \quad (49)$$

and since $Z_1=Z_2$ is assumed

$$Z_T=Z_1\left(\frac{1+2A}{1+A}\right) \quad (50)$$

Calculate the currents, and voltages at bus G

$$I_{a1}=\frac{V_a'}{Z_T}$$

$$I_{a2}=-I_{a1}\frac{Z_0}{Z_2+Z_0}=-I_{a1}\frac{AZ_1}{Z_1(1+A)}=\frac{A}{1+A}\times I_{a1}\angle 180^\circ \quad (51)$$

$$I_{a0}=-I_{a1}\frac{Z_2}{Z_2+Z_0}=-I_{a1}\frac{Z_1}{Z_1(1+A)}$$

$$=\frac{1}{1+A}I_{a1}\angle 180^\circ \quad (52)$$

$$I_a=I_{a1}+I_{a2}+I_{a0}=0 \quad (53)$$

$$I_b=a^2I_{a1}+aI_{a2}+I_{a0}=$$

$$I_{a1}\left(\angle 240^\circ+\frac{A}{1+A}\angle 300^\circ+\frac{1}{1+A}\angle 180^\circ\right)$$

$$=\frac{V_a'}{Z_T(1+A)}(\sqrt{3}\angle 210^\circ+\sqrt{3}A\angle 270^\circ) \quad (54)$$

Similarly,

$$I_c=\frac{V_a'}{Z_T(1+A)}(\sqrt{3}\underline{/150^\circ}+\sqrt{3}A\underline{/90^\circ}) \quad (55)$$

$$I_a-I_b=\frac{V_a'}{Z_T(1+A)}(\sqrt{3}\underline{/30^\circ}+\sqrt{3}A\underline{/90^\circ}) \quad (56)$$

$$I_b-I_c=\frac{V_a'}{Z_T(1+A)}(\sqrt{3}\underline{/270^\circ}+2\sqrt{3}A\underline{/270^\circ}) \quad (57)$$

$$I_c-I_a=\frac{V_a'}{Z_T(1+A)}(\sqrt{3}\underline{/150^\circ}+\sqrt{3}A\underline{/90^\circ}) \quad (58)$$

$$V_{a1}=V_a'-I_{a1}Z_{1S}=V_a'-\frac{V_a'}{Z_T}Z_{1S}=\frac{V_a'}{Z_T}(Z_T-Z_{1S}) \quad (59)$$

$$V_{a2}=-I_{a2}Z_{2S}=\frac{A}{1+A}I_{a1}Z_{1S}=\frac{V_a'}{Z_T}\left(\frac{A}{1+A}Z_{1S}\right) \quad (60)$$

$$V_{a0}=-I_{a0}Z_{0S}=\frac{1}{1+A}I_{a1}Z_{0S}=\frac{V_a'}{Z_T}\left(\frac{1}{1+A}Z_{0S}\right) \quad (61)$$

$$V_a=V_{a1}+V_{a2}+V_{a0}=\frac{V_a'}{Z_T}\left(Z_T+Z_{1S}\left(-1+\frac{A}{1+A}\right)+\frac{Z_{0S}}{1+A}\right)=\frac{V_a'}{Z_T(1+A)}(2AZ_{1S}+Z_{0S}+Z_{1L}(1+2A)) \quad (62)$$

$$V_b=a^2V_{a1}+aV_{a2}+V_{a0}=\frac{V_a'}{Z_T}\left((Z_T-Z_{1S})a^2+\frac{A}{1+A}aZ_{1S}+\frac{1}{1+A}Z_{0S}\right)=\frac{V_a'}{Z_T(1+A)}(AZ_{1S}\underline{/180^\circ}+Z_{0S}+(1+2A)Z_{1L}\underline{/240^\circ}) \quad (63)$$

$$V=aV_{a1}+a^2V_{a2}+V_{a0}=\frac{V_a'}{Z_T}\left((Z_T-Z_{1S})a+\frac{A}{1+A}a^2Z_{1S}+\frac{Z_{0S}}{1+A}\right)=\frac{V_a'}{Z_T(1+A)}(AZ_{1S}\underline{/180^\circ}+Z_{0S}+(1+2A)Z_{1L}\underline{/120^\circ}) \quad (64)$$

The phase-to-phase voltages are derived as before, and separating source and line impedances, they are found to be

$$V_{ab}=\frac{V_a'}{Z_T(1+A)}(3AZ_{1S}+\sqrt{3}(1+2A)Z_{1L}\times\underline{/30^\circ}) \quad (65)$$

$$V_{bc}=\frac{V_a'}{Z_T(1+A)}(\sqrt{3}(1+2A)Z_{1L}\underline{/270^\circ}) \quad (66)$$

$$V_{ca}=\frac{V_a'}{Z_T(1+A)}(3AZ_{1S}\underline{/180^\circ}+\sqrt{3}(1+2A)\times Z_{1L}\underline{/150^\circ}) \quad (67)$$

With the expressions of voltage and current now available, the relay formulas are derived in a manner similar to those in the two preceding sections. The algebra of the operations is a bit more complicated in this case, however, because of the greater complexity of the current expressions, hence the derivations will be given here for the "90° Connection," to illustrate the method.

90° Connection

Relay A: $$Z_D=\frac{V_{bc}}{I_a}=\infty \quad (68)$$

Relay B: $$Z_D=\frac{V_{ca}}{I_b}=\frac{3AZ_{1S}\underline{/180^\circ}+\sqrt{3}(1+2A)Z_{1L}\underline{/150^\circ}}{\sqrt{3}\underline{/210^\circ}+\sqrt{3}A\underline{/270^\circ}} \quad (69)$$

There are two different angular operators in the denominator so that it is impossible to reduce each of them to a real number in the same way that the denominators in previous illustrations were reduced. The best that can be done is to reduce one of the factors to a real term, and the factor chosen is the one not involving A. Inspection shows that if an angle of 150° is added, this will be done, hence multiply both numerator and denominator by $\underline{/150^\circ}$, and cancel out $\sqrt{3}$ from each.

$$Z_D=\frac{\sqrt{3}AZ_{1S}\underline{/330^\circ}+(1+2A)Z_{1L}\underline{/300^\circ}}{1+A\underline{/60^\circ}} \quad (70)$$

Relay C: Z_D

$$=\frac{3AZ_{1S}+\sqrt{3}(1+2A)Z_{1L}\underline{/30^\circ}}{\sqrt{3}\underline{/150^\circ}+\sqrt{3}A\underline{/90^\circ}}\times\frac{\underline{/210^\circ}}{\underline{/210^\circ}}=\frac{\sqrt{3}AZ_{1S}\underline{/210^\circ}+(1+2A)Z_{1L}\underline{/240^\circ}}{1+A\underline{/300^\circ}} \quad (71)$$

Utilizing the above method of derivation yields the formulas for the remaining relay connections, and these are listed in Table I.

Three-Phase Fault

Calculate the currents, and voltages at bus G

$$I_{a1}=\frac{V_a'}{Z_{1S}+Z_{1L}}=\frac{V_a'}{Z_T} \quad (72)$$

$$I_{a2}=I_{a0}=0 \quad (73)$$

$$I_a=I_{a1}=\frac{V_a'}{Z_T} \quad (74)$$

$$I_b=a^2I_{a1}=a^2\frac{V_a'}{Z_T} \quad (75)$$

$$I_c=aI_{a1}=a\frac{V_a'}{Z_T} \quad (76)$$

$$I_a-I_b=\sqrt{3}\frac{V_a'}{Z_T}\underline{/30^\circ} \quad (77)$$

$$I_b-I_c=\sqrt{3}\frac{V_a'}{Z_T}\underline{/270^\circ} \quad (78)$$

$$I_c-I_a=\sqrt{3}\frac{V_a'}{Z_T}\underline{/150^\circ} \quad (79)$$

$$V_a=I_{a1}Z_{1L}=\frac{V_a'}{Z_T}Z_{1L} \quad (80)$$

$$V_b=a^2V_a=\frac{V_a'}{Z_T}Z_{1L}\underline{/240^\circ} \quad (81)$$

$$V_c=aV_a=\frac{V_a'}{Z_T}Z_{1L}\underline{/120^\circ} \quad (82)$$

$$V_{ab}=V_a-V_b=\sqrt{3}\frac{V_a'}{Z_T}Z_{1L}\underline{/30^\circ} \quad (83)$$

$$V_{bc}=V_b-V_c=\sqrt{3}\frac{V_a'}{Z_T}Z_{1L}\underline{/270^\circ} \quad (84)$$

$$V_{ca}=V_c-V_a=\sqrt{3}\frac{V_a'}{Z_T}Z_{1L}\underline{/150^\circ} \quad (85)$$

Because of the symmetry of the 3-phase fault, only one relay formula is needed per connection. These are derived as before, and are listed in Table I.

Appendix II. System Figure 2

Inasmuch as Appendix I has shown the method of derivation of the Z_D formulas for the relay directional elements as well as the method of deriving the currents and voltages this appendix will merely list for reference the currents and voltages involved. This will only be necessary for faults involving ground, because for phase-to-phase and 3-phase faults, Figure 2 is a duplicate of Figure 1. The Z_D formulas are given in Table II.

Phase-A-to-Ground Fault

$$Z_T=Z_1+Z_2+Z_0=2Z_1+Z_0 \quad (86)$$

$$I_{a1}=I_{a2}=I_{a0}=\frac{V_a'}{Z_T} \quad (87)$$

$$I_a=2\frac{V_a'}{Z_T} \quad (88)$$

$$I_b=\frac{V_a'}{Z_T}\underline{/180^\circ} \quad (89)$$

$$I_c=\frac{V_a'}{Z_T}\underline{/180^\circ} \quad (90)$$

$$I_a-I_b=3\frac{V_a'}{Z_T} \quad (91)$$

$$I_b-I_c=0 \quad (92)$$

$$I_c-I_a=3\frac{V_a'}{Z_T}\underline{/180^\circ} \quad (93)$$

$$V_{a1}=\frac{V_a'}{Z_T}(Z_T+Z_{1S}\underline{/180^\circ}) \quad (94)$$

$$V_{a2}=\frac{V_a'}{Z_T}Z_{1S}\underline{/180^\circ} \quad (95)$$

$$V_{a0}=\frac{V_a'}{Z_T}Z_0\underline{/180^\circ} \quad (96)$$

$$V_a=\frac{V_a'}{Z_T}2Z_{1L} \quad (97)$$

$$V_b=\frac{V_a'}{Z_T}(\sqrt{3}Z_{1S}\underline{/270^\circ}+\sqrt{3}Z_0\underline{/210^\circ}+2Z_{1L}\underline{/240^\circ}) \quad (98)$$

$$V_c=\frac{V_a'}{Z_T}(\sqrt{3}Z_{1S}\underline{/90^\circ}+\sqrt{3}Z_0\underline{/150^\circ}+2Z_{1L}\underline{/120^\circ}) \quad (99)$$

$$V_{ab}=\frac{V_a'}{Z_T}(\sqrt{3}Z_{1S}\angle 90^\circ+\sqrt{3}Z_0\angle 30^\circ+2\sqrt{3}Z_{1L}\angle 30^\circ) \quad (100)$$

$$V_{bc}=\frac{V_a'}{Z_T}(2\sqrt{3}Z_{1S}\angle 270^\circ+\sqrt{3}Z_0\angle 270^\circ+2\sqrt{3}Z_{1L}\angle 270^\circ) \quad (101)$$

$$V_{ca}=\frac{V_a'}{Z_T}(\sqrt{3}Z_{1S}\angle 90^\circ+\sqrt{3}Z_0\angle 150^\circ+2\sqrt{3}Z_{1L}\angle 150^\circ) \quad (102)$$

Two-Phase-to-Ground Fault on Phases B and C

$$Z_T=Z_1+\frac{Z_0Z_2}{Z_0+Z_2}=Z_1\frac{1+2A}{1+A} \quad (103)$$

$$A=\frac{Z_0}{Z_1} \text{ and } Z_1=Z_2 \quad (104)$$

$$I_{a1}=\frac{V_a'}{Z_T} \quad (105)$$

$$I_{a2}=\frac{A}{1+A}\frac{V_a'}{Z_T}\angle 180^\circ \quad (106)$$

$$I_{a0}=\frac{1}{1+A}\frac{V_a'}{Z_T}\angle 180^\circ \quad (107)$$

Currents at relay location, bus G, do not involve zero sequence.

Therefore

$$I_a=I_{a1}+I_{a2}=\frac{V_a'}{Z_T}\frac{1}{1+A} \quad (108)$$

$$I_b=\frac{V_a'}{Z_T}\frac{1}{1+A}(1\angle 240^\circ+\sqrt{3}A\angle 270^\circ) \quad (109)$$

$$I_c=\frac{V_a'}{Z_T}\frac{1}{1+A}(1\angle 120^\circ+\sqrt{3}A\angle 90^\circ) \quad (110)$$

$$I_a-I_b=\frac{Va'}{Z_T\ (1+A)}(\sqrt{3}\angle 30^\circ+\sqrt{3}A\angle 90^\circ) \quad (111)$$

$$I_b-I_c=\frac{V_a'}{Z_T\ (1+A)}(\sqrt{3}\angle 270^\circ+2\sqrt{3}A\angle 270^\circ) \quad (112)$$

$$I_c-I_a=\frac{Va'}{Z_T(1+A)}(\sqrt{3}\angle 150^\circ+\sqrt{3}A\angle 90^\circ) \quad (113)$$

$$V_{a1}=\frac{V_a'}{Z_T}(Z_T-Z_{1S}) \quad (114)$$

$$V_{a2}=\frac{A}{1+A}\frac{V_a'}{Z_T}Z_{1S} \quad (115)$$

$$V_{a0}=\frac{1}{1+A}\frac{V_a'}{Z_T}Z_0=\frac{A}{1+A}\frac{V_a'}{Z_T}Z_1 \quad (116)$$

$$V_a=\frac{Va'}{Z_T(1+A)}(3AZ_{1S}+Z_{1L}(1+3A)) \quad (117)$$

$$V_b=\frac{V_a'}{Z_T(1+A)}(Z_{1L}(1\angle 240^\circ+\sqrt{3}A\angle 270^\circ)) \quad (118)$$

$$V_c=\frac{V_a'}{Z_T(1+A)}(Z_{1L}(1\angle 120^\circ+\sqrt{3}A\angle 90^\circ)) \quad (119)$$

$$V_{ab}=\frac{V_a'}{Z_T(1+A)}(3AZ_{1S}+\sqrt{3}(1+2A)Z_{1L}\angle 30^\circ) \quad (120)$$

$$V_{bc}=\frac{V_a'}{Z_T(1+A)}(\sqrt{3}(1+2A)Z_{1L}\angle 270^\circ) \quad (121)$$

$$V_{ca}=\frac{V_a'}{Z_T(1+A)}(3AZ_{1S}\angle 180^\circ+\sqrt{3}(1+2A)Z_{1L}\angle 150^\circ) \quad (122)$$

Appendix III. System Figure 3

In this case, a delta star connected transformer bank is interposed between the relay location and the fault. The phase relationships assumed are that the star side voltages x, y, z, (high voltage assumed) are 30° leading the delta voltages a, b, c (low voltage assumed). Calculations are made initially from the star connected side, but it must be remembered that in passing through the transformer bank to the relay location, the positive sequence currents and voltages are rotated 30° lagging, the similar negative sequence components are rotated 30° leading, and the zero sequence current and voltage disappear.

In line with the policy established in Appendix II, only the expressions for the necessary currents and voltages will be shown here, remembering that the Z_D formulas are given in Table 3. Also, V_x' is used for the source voltage in this case rather than V_a' because of the different phase terminology used to identify the side of the transformer bank away from the relay.

Phase-to-Phase Fault on Phases YZ

$$Z_1=Z_{1S}+Z_{1TR}+Z_{1L}=Z_2 \quad (123)$$

$$Z_T=Z_1+Z_2=2Z_1 \quad (124)$$

$$I_{a1}=\frac{V_x'}{2Z_1}\angle 330^\circ \quad (125)$$

$$I_{a2}=\frac{V_x'}{2Z_1}\angle 210^\circ \quad (126)$$

$$I_a=\frac{V_x'}{2Z_1}\angle 270^\circ \quad (127)$$

$$I_b=\frac{V_x'}{2Z_1}\angle 270^\circ \quad (128)$$

$$I_c=\frac{V_x'}{Z_1}\angle 90^\circ \quad (129)$$

$$I_a-I_b=0 \quad (130)$$

$$I_b-I_c=\frac{3V_x'}{2Z_1}\angle 270^\circ \quad (131)$$

$$I_c-I_a=\frac{3V_x'}{2Z_1}\angle 90^\circ \quad (132)$$

$$V_{a1}=\frac{V_x'}{2Z_1}(2Z_1\angle 330^\circ+Z_{1S}\angle 150^\circ) \quad (133)$$

$$V_{a2}=\frac{V_x'}{2Z_1}Z_{1S}\angle 30^\circ \quad (134)$$

$$V_a=\frac{V_x'}{2Z_1}(2Z_1\angle 330^\circ+Z_{1S}\angle 90^\circ) \quad (135)$$

$$V_b=\frac{V_x}{2Z_1}(2Z_1\angle 210^\circ+Z_{1S}\angle 90^\circ) \quad (136)$$

$$V_c=\frac{V_x'}{2Z_1}(2Z_1\angle 90^\circ+2Z_{1S}\angle 270^\circ) \quad (137)$$

$$V_{ab}=\sqrt{3}V_x' \quad (138)$$

$$V_{bc}=\frac{\sqrt{3}V_x'}{2Z_1}(2Z_1\angle 240^\circ+\sqrt{3}Z_{1S}\angle 90^\circ) \quad (139)$$

$$V_{ca}=\frac{\sqrt{3}V_x'}{2Z_1}(2Z_1\angle 120^\circ+\sqrt{3}Z_{1S}\angle 270^\circ) \quad (140)$$

Phase-to-Ground Fault on Phase X

$$Z_T=Z_1+Z_2+Z_0=2Z_1+Z_0 \quad (141)$$

$$Z_1=Z_{1S}+Z_{1TR}+Z_{1L} \quad (142)$$

$$Z_0=Z_{0TR}+Z_{0L} \quad (143)$$

$$I_{a1}=\frac{V_x'}{Z_T}\angle 330^\circ \quad (144)$$

$$I_{a2}=\frac{V_x'}{Z_T}\angle 30^\circ \quad (145)$$

$$I_{a0}=0 \quad (146)$$

$$I_a=\sqrt{3}\frac{V_x'}{Z_T} \quad (147)$$

$$I_b=\sqrt{3}\frac{V_x'}{Z_T}\angle 180^\circ \quad (148)$$

$$a_c=0 \quad (149)$$

$$I_a-I_b=2\sqrt{3}\frac{V_x'}{Z_T} \quad (150)$$

$$I_b-I_c=\sqrt{3}\frac{V_x'}{Z_T}\angle 180^\circ \quad (151)$$

$$I_c-I_a=\sqrt{3}\frac{V_x'}{Z_T}\angle 180^\circ \quad (152)$$

$$V_{a1}=\frac{V_x'}{Z_T}(Z_T\angle 330^\circ+Z_{1S}\angle 150^\circ) \quad (153)$$

$$V_{a2}=\frac{V_x'}{Z_T}Z_{1S}\angle 210^\circ \quad (154)$$

$$V_a=\frac{V_x'}{Z_T}(Z_T\angle 330^\circ+\sqrt{3}Z_{1S}\angle 180^\circ) \quad (155)$$

$$V_b=\frac{V_x'}{Z_T}(Z_T\angle 210^\circ+\sqrt{3}Z_{1S}) \quad (156)$$

$$V_c=\frac{V_x'}{Z_T}Z_T\angle 90^\circ \quad (157)$$

$$V_{ab}=\frac{V_x'}{Z_T}(\sqrt{3}Z_T+2\sqrt{3}Z_{1S}\angle 180^\circ) \quad (158)$$

$$V_{bc}=\frac{V_x'}{Z_T}(\sqrt{3}Z_T\angle 240^\circ+\sqrt{3}Z_{1S}) \quad (159)$$

$$V_{ca}=\frac{V_x'}{Z_T}(\sqrt{3}Z_T\angle 120^\circ+\sqrt{3}Z_{1S}) \quad (160)$$

Two-Phase-to-Ground Fault on Phases YZ

$$Z_1=Z_{1S}+Z_{1TR}+Z_{1L}=Z_2 \quad (16$$

$$Z_0=Z_{0TR}+Z_{0L}=AZ_1 \quad (162)$$

$$Z_T = Z_1 + \frac{Z_0 Z_2}{Z_0+Z_2} = Z_1\frac{1+2A}{1+A} \quad (163)$$

$$I_{a1} = \frac{V_x'}{Z_T}\underline{/330°} \quad (164)$$

$$I_{a2} = \frac{V_x'}{Z_T}\frac{A}{1+A}\underline{/210°} \quad (165)$$

$$I_{a0} = 0 \quad (166)$$

$$I_a = \frac{V_x'}{Z_T(1+A)}(1\underline{/330°} + A\underline{/270°}) \quad (167)$$

$$I_b = \frac{V_x'}{Z_T(1+A)}(1\underline{/210°} + A\underline{/270°}) \quad (168)$$

$$I_c = \frac{V_x'}{Z_T(1+A)}(1\underline{/90°} + 2A\underline{/90°}) \quad (169)$$

$$I_a - I_b = \sqrt{3}\frac{V_x'}{Z_T(1+A)} \quad (170)$$

$$I_b - I_c = \frac{V_x'}{Z_T(1+A)}(\sqrt{3}\underline{/240°} + 3A\underline{/270°}) \quad (171)$$

$$I_c - I_a = \frac{V_x'}{Z_T(1+A)}(\sqrt{3}\underline{/120°} + 3A\underline{/90°}) \quad (172)$$

$$V_{a1} = \frac{V_x'}{Z_T}(Z_T\underline{/330°} + Z_{1S}\underline{/150°}) \quad (173)$$

$$V_{a2} = \frac{V_x'}{Z_T}\left(\frac{A}{1+A}Z_{1S}\underline{/30°}\right) \quad (174)$$

$$V_a = \frac{V_x'}{Z_T(1+A)}(\sqrt{3}AZ_{1S} + (1+2A)(Z_{1TR}+Z_{1L})\underline{/330°}) \quad (175)$$

$$V_b = \frac{V_x'}{Z_T(1+A)}(\sqrt{3}AZ_{1S}\underline{/180°} + (1+2A)(Z_{1TR}+Z_{1L})\underline{/210°}) \quad (176)$$

$$V_c = \frac{V_x'}{Z_T(1+A)}((1+2A)(Z_{1TR}+Z_{1L})\underline{/90°}) \quad (177)$$

$$V_{ab} = \frac{V_x'\sqrt{3}}{Z_T(1+A)}(2AZ_{1S} + (1+2A)(Z_{1TR}+Z_{1L}) \quad (178)$$

$$V_{bc} = \frac{V_x'\sqrt{3}}{Z_T(1+A)}(AZ_{1S}\underline{/180°} + (1+2A)(Z_{1TR}+Z_{1L})\underline{/240°}) \quad (179)$$

$$V_{ca} = \frac{V_x'\sqrt{3}}{Z_T(1+A)}(AZ_{1S}\underline{/180°} + (1+2A)(Z_{1TR}+Z_{1L})\underline{/120°}) \quad (180)$$

Three-Phase Fault

$$Z_T = Z_1 = Z_{1S} + Z_{1TR} + Z_{1L} \quad (181)$$

$$I_{a1} = \frac{V_x'}{Z_1}\underline{/330°} \quad (182)$$

$$I_{a2} = I_{a0} = 0 \quad (183)$$

$$I_a = I_{a1} = \frac{V_x'}{Z_1}\underline{/330°} \quad (184)$$

$$I_b = \frac{V_x'}{Z_1}\underline{/210°} \quad (185)$$

$$I_c = \frac{V_x'}{Z_1}\underline{/90°} \quad (186)$$

$$I_a - I_b = \frac{V_x'}{Z_1}\sqrt{3} \quad (187)$$

No other "delta" currents are needed.

$$V_a = I_a(Z_{1TR}+Z_{1L}) = \frac{V_x'}{Z_1}(Z_{1TR}+Z_{1L})\underline{/330°} \quad (188)$$

$$V_b = \frac{V_x'}{Z_1}(Z_{1TR}+Z_{1L})\underline{/210°} \quad (189)$$

$$V_c = \frac{V_x'}{Z_1}(Z_{1TR}+Z_{1L})\underline{/90°} \quad (190)$$

$$V_{ac} = V_a - V_c = \frac{V_x'}{Z_1}(Z_{1TR}+Z_{1L})\sqrt{3}\underline{/300°} \quad (191)$$

$$V_{bc} = V_b - V_c = \frac{V_x'}{Z_1}(Z_{1TR}+Z_{1L})\sqrt{3}\underline{/240°} \quad (192)$$

No other voltages are needed.

Discussion

Myron A. Bostwick (Portland General Electric Company, Portland, Oreg.): I wish to compliment Mr. Sonnemann for his excellent analysis of single-phase directional element connections. He has used a simple, readily understood approach to the problem, and provides more definite limits for successful operation than have been published previously. A glance at the many pages of calculations provides an indication of the thoroughness of his investigation.

Fortunately it is not necessary to dig through these voluminous calculations to use the information that has been provided. The vector diagrams tell the whole story, when considered in terms of the particular relay characteristics and connections. It also is essential to consider details of the power circuit as Mr. Sonnemann has indicated.

The need of this is emphasized by consideration of the diagrams, as described in the section on Summary of Results. The author has indicated that there are "three cases where the current falls in the contact opening zone by being either more than 45 degrees lagging or more than 135 degrees leading." He then discusses three conditions that may cause undesirable operation. In this discussion he fails to mention two conditions (the B relay, 90-degree connection, for a phase A-to-ground fault on system Figure 2, in table VI and the phase A relay, 90-degree connection, in Table VII for a Y-Z fault in system Figure 3) that also cause contact opening torque. Discussion of these was apparently omitted because the relays will not trip incorrectly on the reversed flow of current to an external fault. I think it desirable to have consideration of these relays included in the discussion for future reference and suggest that it be added to the paper.

The analysis has clinched my opinion that the 90-degree connection should be used generally in all applications, except those noted as being subject to incorrect operation. Individual study of those installations may then be required. In closing these remarks, I wish to thank Mr. Sonnemann for his clear analysis of the problem, and the tremendous amount of work that was required to present it.

W. K. Sonnemann: Mr. Bostwick has picked out two conditions which should have been discussed in the original paper. In the fourth paragraph preceding the "Conclusions," the reference should be to five cases where the current falls in the contact opening zone rather than just three. As a matter of fact, all of the vector diagrams were studied individually for their implications at the time of their original derivation, including these two. At the time of sifting and sorting all of the work which had been done in order to prepare the paper, however, these two were missed. The author regrets that they were missed, and thanks Mr Bostwick for calling attention to them.

In the first case, relay B, 90-degree connection, phase-A-to-ground fault, system Figure 2, there is the possibility of incorrect tripout for relays located at certain other points than that illustrated in Figure 2. For example, relays in the source circuit at bus G, as well as relays on the left-hand side of any bus interposed between the bus G and the fault, will "see" the vector relationships of Table VI line 1 in reverse, hence it is apparent that the possibility of erroneous tripout for relay B should be examined further. It is in cases such as this that it is well to bear in mind that the vector diagrams show extreme conditions which may be more theoretical than practical, and that an examination of the system constants may show the trouble to be nonexistent for a particular system. The formula for relay B under discussion is:

$$Z_D = \sqrt{3}[Z_{1S}\underline{/270°} + (Z_0 + 2Z_{1L})\underline{/330°}]$$

The limit condition of the current lagging by 60 degrees was obtained in this case by assuming that the zero sequence impedance is the governing factor through being large enough to make the other factors negligible, and that it has an angle of 90 degrees. The 90-degree angle is not too bad an assumption if the fault occurs at the far end of the line at the grounding bank location provided that the bank is not grounded through a resistor. If the fault is at the far end of the line, however, the impedance of the line is not necessarily negligible, and whether or not the source impedance is negligible by comparison depends upon the system. Assume, for example, that the line impedance is negligible, and take Z_0 at 1.00 $\underline{/90°}$ for reference. If Z_{1S} is assumed at 90 degrees, it is found that it requires a scalar value of only 0.366 to bring the Z_D value to a 45-degree angle, the border line condition. Thus, if Z_{1S} exceeds 36.6 per cent of Z_0, the problem disappears for relays having a 45-degree characteristic. If Z_{1S} has a smaller angle than 90 degrees, a slightly greater value will be required. If the transformer is grounded through a resistor, the characteristic angle of Z_0 approaches zero degrees, and the current approaches the 30-degree leading position.

The second case mentioned by Mr. Bostwick covering relay A, 90-degree connection, YZ fault, system Figure 3 is similar to the first in that relays in source circuits at bus G require scrutiny. The most severe condition occurs when the fault is right at the line terminals of the transformer so that Z_{1L}, assumed to have a smaller angle than 90 degrees, is zero. The angle of the transformer impedance Z_{1TR} is near 90 degrees, however, and dependence must be placed on the source impedance being relatively large in order to reduce the angle of Z_D to less than 45 degrees. If there are several source circuits connected to the bus, the condition will be materially aggravated for the relays on these source circuits by virtue of the multiplying effect of the current from the other sources on the apparent value of Z_{1TR} as seen, in reverse, by any one set of relays on a source circuit. It is conceivable that there will be systems wherein the 90-degree connection, with the 45-degree phase shifter, will be unsatisfactory. In such cases, the solution is to modify the 45-degree phase shifter if this does not cause trouble from one of the other limits, or else use one of the two 60-degree connections.

Local Wire Video Television Networks

C. N. NEBEL
NONMEMBER AIEE

A satisfactory local video distribution system is necessary to interconnect television studios, transmitters, and pick-up points within a city and to connect them to the coaxial cable and microwave intercity television networks.[1,2] Wire extension circuits also are necessary to distribute video signals from local microwave circuits which are used to pick up remote program material. This is necessary because space on a broadcaster's premise is often unavailable and undesirable from a system maintenance point of view for the operation of remote microwave equipments. A local video distribution system has been developed for the Bell Telephone System utilizing existing telephone facilities as much as practicable. Backbone routes in local areas use newly developed low loss shielded video cable pairs. In many cases these are supplemented by end links of existing paper pair cables.

Equipment is provided for the equalization and amplification of the video signals transmitted over wire circuits. This equipment consists of a transmitting terminal, an intermediate repeater with cable equalizers, and a receiving terminal. Video repeating coils (line transformers) are sometimes used at the terminals for direct connections to the cable pairs. A clamper also is used at the receiving terminal as a stabilizing element for low-frequency circuit noise and for restoring the low-frequency signal components which are removed by the terminal repeating coils.

The intermediate repeater contains four blocks of gain in the form of input and output amplifiers and two amplifier gain networks. The equalization for the cable loss is obtained by utilizing the gain-frequency characteristics of the two amplifier gain networks and of the equalizers which are connected between the two amplifiers. The equalization and gain is distributed throughout the system so as to obtain a reduction in cable and amplifier noise. Increased high-frequency noise suppression is obtained by means of signal predistortion, that is by transmitting the low-level high-frequency signal components over the system at a higher than normal level and restoring them to normal level at the receiving end of the circuit.

It is the purpose of this paper to describe these video transmission facilities more fully and to show the results obtained with some typical circuit applications.

Paper 50-234, recommended by the AIEE Wire Communication Systems Committee and approved by the AIEE Technical Program Committee for presentation at the AIEE Fall General Meeting, Oklahoma City, Okla., October 23-27, 1950. Manuscript submitted July 31, 1950; made available for printing August 29, 1950.

C. N. Nebel is with the Bell Telephone Laboratories Inc., Murray Hill, N. J.

Characteristics of Cable Plant

The cable plant in the local exchange areas available for video transmission consists of coaxial cables, existing interoffice trunk and subscriber paper insulated cables, and shielded polyethylene string and tape insulated cables.

Coaxial Cables

The first 4-megacycle video transmission circuits were used for relatively short studio-transmitter connections. Special installations of 0.268-inch coaxial cables with specially designed equalizers were provided to care for these cases. These were very special cases where the 4-megacycle loss was less than 20 decibels requiring no intermediate gain in the cables. Although they were short circuits, they had the disadvantage that ground currents induced into the outer coaxial conductor by neighboring power circuits caused serious interference. Although it could be balanced out by special means on short circuits, it was found impracticable to employ long unbalanced coaxial circuits in a large video transmission plant. Balanced pair cables provide a natural balance of 75 decibels or more to these interfering power frequencies and are, therefore, much more suitable for use in the development of a large video plant facility.

Existing Cable Plant

A wide variety of balanced circuits are to be found in the existing plant in the form of paper-insulated interoffice trunks and subscriber cable pairs. These existing facilities were fairly extensively used in the initial stages of this video plant development. It will be interesting at this point to examine the limitations of these circuits for video transmission.

There are four principal sources of noise interference on balanced transmission circuits. First are the voltages induced between the line conductors and ground from neighboring power systems. These act upon unbalances between the line conductors and ground to produce components in the metallic circuit. Second are impulse-type noises arising from cross-induction from message-circuit pairs in the cable. These originate in the many switching operations occurring in

Table I. Attenuation of Line Facilities for Video Transmission

Paper Pair Cables, Gauge	Attenuation in Decibels at 4 Megacycles: 1,000 Feet	Per Mile
10	4.7	25.0
13	5.7	30.0
16	6.9	36.0
19 CNB	10.0	53.0
22	13.0	68.0
24	16.0	84.0
26	18.0	95.0
Shielded Video Cables		
Number 16 *PSV* spiral shield	3.41	18.0
Number 16 *PSV* longitudinal shield	3.22	17.0
754*C* double braid	5.0	27.5
Coaxial Carrier Cables		
0.268-inch longitudinal shield		10.9
0.375-inch longitudinal shield		7.7
Office Cabling		
Paired—754B	5.2	
Coaxial—724	4.63	